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Domino effects within a chemical cluster: A game-theoretical modeling approach by using Nash-equilibrium

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ABSTRACT

Every company situated within a chemical cluster faces domino effect risks, whose magnitude depends on every company's own risk management strategies and on those of all others. Preventing domino effects is therefore very important to avoid catastrophes in the chemical process industry. Given that chemical companies are interlinked by domino effect accident links, there is some likelihood that even if certain companies fully invest in domino effects prevention measures, they can nonetheless experience an external domino effect caused by an accident which occurred in another chemical enterprise of the cluster. In this article a game-theoretic approach to interpret and model behaviour of chemical plants within chemical clusters while negotiating and deciding on domino effects prevention investments is employed.

1. Introduction

In the (petro-)chemical industry, economies of scale, environmental factors, social motives and legal requirements often lead companies to 'cluster'. Therefore, chemical plants are most often physically located in groups and are rarely located separately. These clusters of chemical plants consist of atmospheric, cryogenic and pressurized storage tanks, large numbers of production installation equipment, and numerous pipelines for the transportation of hazardous chemicals. In and around such clusters, dangerous goods are transported in large volumes using pipelines, trucks, ship, barges and trains. Due to the rapid development of chemical technology, there is a continuous growth of ever more complex installations with more extreme and critical process conditions. The incidence and the severity of accidents also tend to increase [1].

In fact, three kinds of accidents can be distinguished in chemical plants: those that happen to individuals, those that happen on an organizational scale and those that happen to clusters of companies. Reason [2] indicates that individual accidents are by far the largest in number and that organizational accidents are comparatively rare, although often much more serious. However, Reason [2] does not mention the potentially most catastrophic type of accidents, i.e., multiple-plant or cluster-related accidents, probably because of the extremely low rate at which they occur. Regrettably, such accidents do occur and often with disastrous consequences. Cluster accidents can be related to linked production and/or linked delivery of services, as well as to cross-company accidents or so-called external domino effects. This article focuses on prevention investments concerning organizational accidents and inter-organizational accidents.

'Domino risk' is a term by which the potential for an escalating interaction between groups of chemical installations in the event of an accident at one of the installations is connoted. The term *domino effect* thus denotes a 'chain of accidents'. This mechanism is also referred to as 'escalation', 'interaction' or 'knock-on'. According to the definition provided by Delvosalle [3], a domino effect implies a cascade of accidents (so-called domino events) in which the consequences of a previous accident are increased by the following one(s), spatially as well as temporally, leading to a major accident. Hence, a domino effect implies a primary accident concerning a primary installation, inducing one or more secondary accident(s). The latter accident(s) concern either the primary installation (i.e., a temporary domino effect) or a secondary installation(s) (i.e., a spatial domino effect). The secondary accident(s) must be a major one(s) and must extend the damages of the primary accident. It is useful to categorize domino effects into two types: internal domino effects and external domino effects. Internal effects happen inside the boundaries of the plant where the domino accident originates, as opposed to external domino effects which happen outside

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the boundaries of the plant where the domino accident originates (Reniers et al. [4]).

Available reports on domino effects (e.g. Kletz [5] and Pietersen [6,7]) describe multiple-accident events which have taken place in the past. The worst such accident – in terms of death toll – occurred in Mexico City on November 19, 1984. It was an external domino effect involving three companies: the PEMEX plant (where the accident originated), the Unigas plant, and the Gasomatico plant. The accident claimed 650 lives and injured approximately 6400 people. Another external domino accident took place on 11 December 2005 and is known as the Buncefield disaster (Hemel Hempstead, United Kingdom) leading to 43 injuries. The main companies involved were Total and Texaco, the owners of the joint venture Hertfordshire Oil Storage Terminal which took up the majority of the space at the Buncefield complex. This accident is said to be the largest fire accident of peacetime Europe and caused huge property damage losses. Other well-known (internal) domino accidents include Beek (the Netherlands) in 1975 (14 fatalities, 104 injuries) and Vishakhapatnam (India) in 1997 (60 fatalities). It is obvious that domino risks or the risks associated with domino effects have a very high destruction potential.

Although the consequences of an accident with domino effects can clearly be very serious, this phenomenon has so far received not the deserved academic attention. The reason for this rather strange observation is twofold. On one hand, mathematical modelling of domino effects is highly complex. On the other hand, the probability of domino accident events is very low. In order to assess domino effect consequences, deterministic models have to be used in combination with probabilistic models. The main problem of deterministic modelling arises from the transient nature of events. The difficulties in applying probabilistic models are due to the fact that the original input data for the probabilistic analyses are often missing. Frequency of occurrence usually includes occurrences caused by external events, such as domino effects. When the largest external effect is (also) modelled, the result is an accurate representation of the total risk. This can and is even done for combined toxic and flammable effects (e.g. Ale and Uijt de Haag [8]). However this does not take into account the plants' own damages, which is the subject here.

Managing domino effect prevention measures is particularly interesting from a game-theoretic viewpoint because prevention investments concerning such accidents involve different companies (and thus different decision makers) situated within a chemical cluster. Hence, multi-company safety efforts and safety decisions influence preventing these types of accidents. Therefore, risks possibly leading to domino effects at a chemical company not only depend on the company's own decisions (e.g. on which preventive measures to take), but also on the decisions of other chemical plants situated within the chemical cluster in which the company is situated. Expectations and perceptions about other companies' decisions will influence investments in domino prevention and as a result the socio-economic outcome might be suboptimal for every company within the cluster. This situation of domino prevention decision-making within a chemical cluster can be modeled as what is called a 'game' and – by solving the game – give conditions for a win–win situation or a so-called Nash-equilibrium where every company in the cluster wins by investing in domino effects prevention. Moreover, this method helps to investigate when (under what conditions and influencing factors) it is possible to tip a non-investing equilibrium into an investing one.

Although it is somewhat unfortunate that the word 'game' comes anywhere near the field of safety, 'Game Theory' is a mathematically oriented discipline within economic sciences investigating strategic choices of the players of the game (which are, e.g. organizations) and the choices' payoffs.

The next section theoretically discusses the game which can be associated with domino effects prevention in a chemical cluster and formulates the research question. Section 3 mathematically models the Domino Effect Game. In Section 4 a three-company example is provided assuming theoretical figures, thereby illustrating the practical usefulness and the potential implications of our suggested model. Section 5 concludes this article.

2. Research question

Domino risks are risks whose consequences depend on a company's own domino risk management strategies and domino risk prevention measures and on those of other companies. Hence, to determine the steps that need to be taken in a chemical cluster for socio-economically optimizing the existing situation as regards domino effects prevention, an investigation is required to identify how single chemical plants manage their domino risks where there is likelihood that even if they have decided to invest in domino prevention measures, they might be harmed due to other companies not following suit. Our research is aimed at predicting the domino prevention outcome of a situation where all the companies make independent decisions on whether to invest in domino effects prevention or not, but are at the same time aware of the strategic domino prevention decisions (to 'invest' or to 'not invest') made by others.

Concerning external domino risks, inefficient, insufficient, inadequate or even ineffective decisions of one company in a chemical cluster can have devastating impacts on other companies of the cluster. These so-called 'negative externalities' are an important characteristic in what we call the "Domino Effect Game" (DE Game). The return to a chemical plant's investments in domino effects prevention depends on the domino effects prevention actions of other chemical plants from the cluster in which the plant is situated. If other companies do invest, then investing is more attractive, and if they do not, then it is less interesting. Hence, the DE Game exhibits 'strategic complementarities' (Milgrom and Roberts [9]).

Following Heal and Kunreuther [10], the DE Game is classified as a game of partial protection with negative externalities, whereby 'externalities' are defined as possible effects that one company can have on another company. Domino effects within one company can have catastrophic effects (i.e., 'negative externalities') on the whole chemical cluster.

It should be noted that a company's decision to invest in domino prevention for decreasing its own domino risks also decreases the domino risks experienced by other companies within the chemical cluster. Hence, the more that companies invest in domino effects prevention, the lower are the negative externalities in the system. This game can have one or multiple so-called 'Nash equilibria'. The Nash equilibrium concept embodies two requirements (Vega-Redondo [11]): (i) players' strategies[1] must be a best response (i.e., the strategies should maximize the players' respective payoffs or should minimize their respective costs), given some well-defined beliefs about the strategies adopted by the other players, and (ii) the beliefs held by each player must be an accurate ex ante prediction of the strategies actually played by the other players.

For the case where there are two Nash equilibria – either all companies invest in domino effects prevention or none of the companies do – the possibility of so-called 'tipping' exists, indicating that inducing some companies to invest in domino effects prevention will lead others to do so as well. If two Nash equilibria are

[1] It should be noted that the strategies in the DE Game for a player can be either to invest in domino effects prevention (*I*) or not to invest in domino effects prevention (*NI*).

involved, the socio-economic optimal solution will be for every plant within the cluster to invest in domino effects prevention.

When there is only a single Nash equilibrium, the domino effects prevention investment choices made by every individual chemical facility will also be socio-economically optimal in some situations. The case where the costs are sufficiently low so that each company wants to invest in domino effects prevention, even if all the other companies did not incur these costs, is a straightforward example of such a situation. If domino effects prevention investments would appear to be very high to every company relative to their potential benefits, then it might be efficient for no company to incur them. In industrial practice, due to the extremely low probabilities of domino effects occurring, many chemical plants belonging to a cluster are not inclined to invest in far-reaching domino effects prevention measures, although it would be better for society if some companies, or in fact all of them, did engage in more extensive prevention measures.

Our research question is therefore to investigate the possibility of tipping the Nash equilibria from a state of 'no investment' to one of 'universal investment' (by all companies) as regards domino effects prevention in a chemical cluster.

3. The domino effects game model

Game theory is the theory of independent and interdependent decision making. Multi-person games of strategy are games involving three or more players, each of whom has partial control over the outcome. It is obvious that such games potentially involve coalitions. The DE Game is a partially cooperative multi-person game in which coalition-forming is allowed and even essential. Let a 'critical coalition' be a coalition of chemical companies where a change from 'not investing in domino effects prevention' to 'investing in domino effects prevention' by its members will induce all non-members of the coalition to follow suit.

Consider x chemical companies composing a chemical cluster $\{x\}$. Let the companies be indexed by i. Every company is characterized by (i) the probability P_i that company i's actions (or lack of actions) lead to a direct loss L_i (caused by internal domino effects), and (ii) the investment in domino effects prevention at a cost c_i which leads for company i to avoidance of direct loss with certainty (it should be noted that c_i can thus be interpreted as a 'hypothetical benefit' of avoiding domino effects by investments in prevention). Furthermore, every company i has a discrete strategy, S_i, that can take as values either I or NI, representing investing in domino effects prevention (I) and not investing in domino effects prevention (NI), respectively. If company i incurs a direct loss, then this may also affect other companies' outcomes. If company i does not incur a direct loss then it will have no negative impact on other companies. The loss to other companies (caused by external domino effects) is considered in this paper as "an indirect impact". Let $l_i(\{y\},S_i)$ then be the expected indirect loss to chemical plant i when it follows strategy S_i and the companies in the chemical sub-cluster $\{y\}$ are the only ones from the chemical cluster $\{x\}$ investing in domino effects prevention ($y \leq x$). It should be noted that the model is conceptualized such that a company who has invested in domino effects prevention cannot cause an indirect impact on others.

Furthermore, if every other company than company i invests in domino effects prevention, then company i cannot suffer indirect impacts (i.e., impacts from other companies). In other words, if $\{y\} = \{1,2,\ldots,i-1,i+1,\ldots,x\}$ then $l_i(\{y\},S_i) = 0$, independent of the situation where i invests or does not invest in domino effects prevention.

Assume that company i invests in domino effects prevention. Furthermore, assume that chemical companies belonging to the sub-cluster $\{y\}$ also invest in domino effects prevention. Then, the expected loss to company i is $c_i + l_i(\{y\},I)$, whereby the second term in this formula is the expected cost of indirect impacts (consequences) imposed by companies belonging to $\{x\}$ who do not invest in domino effects prevention. The expected loss of company i not investing in domino effects prevention can be expressed as:

$$P_i L_i \prod_{j \neq i, j \in \{x/y\}} (1 - P_j) + (1 - P_i) \cdot l_i(\{y\}, NI)$$

Hence, direct (internal) domino effects (the first term in the latter expression) are conditioned on the non-occurrence of indirect losses. Indirect effects (the second term in the latter expression) are conditioned on direct losses not occurring. These conditions result from the fact that a chemical installation can only explode or be destroyed once or that internal and external domino effects do not originate at the same time.

A chemical company i is indifferent between investing and not investing in domino effects prevention if:

$$c_i + l_i(\{y\}, I) = P_i L_i \prod_{j \neq i, j \in \{x/y\}} (1 - P_j) + (1 - P_i) \cdot l_i(\{y\}, NI)$$

Hence, we can define the cost of investment in domino effects prevention at which company i is indifferent:

$$\tilde{c}_i(\{y\}) = P_i L_i \prod_{j \neq i, j \in \{x/y\}} (1 - P_j) + (1 - P_i) \cdot l_i(\{y\}, NI) - l_i(\{y\}, I)$$

If $c_i > \tilde{c}_i(\{y\})$, then company i will not invest in domino effects prevention, and vice versa.

Furthermore, if a chemical company within a chemical cluster $\{x\}$ decides to invest in domino effects prevention, the sub-cluster $\{y\}$ is increased by one unit. As a result, the expected indirect loss, $P_i l_i(\{y\},NI)$, decreases. Following, $\tilde{c}_i(\{y\})$ increases in $\{y\}$, since more chemical companies investing in domino effects prevention leads to lower expected indirect losses. Thus, the maximum cost at which domino effects prevention investments are justified, increases with every company deciding to invest in such prevention.

A Nash equilibrium for the DE Game is a set of pure strategies $S_1,\ldots,S_x$ such that (i) $S_i = I$, $\forall i \in \{y\}$ (which may be empty), (ii) if $\tilde{c}_i(\{y\}) > c_i$ then $S_i = I$, (iii) if $\tilde{c}_i(\{y\}) < c_i$ then $S_i = NI$, and (iv) company i is indifferent between I and NI, then $\tilde{c}_i(\{y\}) = c_i$.

A Nash equilibrium exists for this kind of game-theoretic problem (i.e., partial protection with negative externalities) (Heal and Kunreuther [10] and Milgrom and Roberts [9]). There may be equilibria where all chemical companies invest in domino effects prevention, those where none do, and asymmetric pure strategy equilibria where some plants invest and others do not. If there are two equilibria, one with all enterprises not investing and the other with everyone investing in domino effects prevention, then it should be investigated in how to possibly tip the socially suboptimal equilibrium (NI, NI, ..., NI) to a socially optimal equilibrium (I, I, ..., I). Hence, it is examined how the DE Game with two (or more) equilibria may be tipped by a change in the strategy choices of a small number of players.

Let (NI, NI, ..., NI) be a Nash equilibrium. A Tipping Inducing Sub-Cluster (TISC) for this equilibrium is a set $\{z\}$ of chemical companies such that if $S_i = I$, $\forall i \in \{z\}$, then $\tilde{c}_j(\{z\}) \geq c_j$, $\forall j \notin \{z\}$. In other words, a Tipping Inducing Sub-Cluster is a sub-cluster with the property that if all chemical plants belonging to that sub-cluster decide to invest in domino effects prevention, then for all other companies belonging to the entire chemical cluster the best strategy is also to invest in such prevention. A 'Minimum TISC' is a TISC of which no subset is also a TISC, indicating that all companies in the Minimum TISC are required to tip the other (non-investing) companies into a domino effects investment strategy. Furthermore, since there can be several Minimum TISCs and we are only interested in the one containing the smallest number of companies, we let the 'Smallest

Number Minimum TISC' be a Minimum TISC where no other TISC includes fewer companies. Assume that the change in the expected indirect loss to plant *i*, who does not invest in domino effects prevention, when company *j* joins the set $\{y\}$ of companies who have already invested in domino effects prevention, is:

$$l_i^j(\{y\}, NI) = l_i\left(\left\{y/j\right\}, NI\right) - l_i(\{y\}, NI) \geq 0, \forall j \in \{y\}$$

Heal and Kunreuther [9] then prove that a Smallest Number Minimum TISC is easily characterized. They further indicate (in general terms) that an equilibrium where no company invests (e.g. in domino effects prevention) may be converted to one with full investment by persuading a subset of the companies to change their policies. Hence, the least expensive way of changing equilibrium is providing incentives for a Tipping Inducing Sub-Cluster to change its behavior, which will then tip the entire cluster.

Hence, a TISC does exist in theory. In order to help clear understanding, the next section provides an illustrative example of how such tipping might occur in a chemical cluster consisting of three companies. First, the illustrative example is given in general terms and at the end, numerical values are used to show the DE Game's validity and its potential in a real industrial setting.

4. An illustrative example

For simplicity, consider a cluster of three companies 1, 2 and 3. Investigation is conducted whether it is possible, and under what conditions, for one company by changing its strategy from *NI* to *I*, to tip the other two companies to change their strategies as well from *NI* to *I*.

Let the factor $P_{i,j}$ represent the probability that a domino event will occur in plant *j* caused by an event which takes place in plant *i* (in other words, $P_{i,j}$ is the likelihood of an external domino effect from company *i* to company *j*). If $i=j$, then the factor expresses the probability for an internal domino effect in company *i*. Every company can decide either to invest in domino effects prevention at a cost c_i (strategy *I*) or not to invest (strategy *NI*). If a domino effect takes place, the loss to company *i* equals L_i. For simplicity, domino effects prevention measures are assumed to be completely effective. Hence, if domino effects prevention investments are made in company *i*, no domino effect can originate from company *i*.

To investigate whether it is possible in the three-company case study for one company to tip the other two companies into changing their strategies, the existing Nash equilibria need to be determined, and the conditions under which strategies are dominant have to be established. Therefore, this paper includes the cost matrices (i.e., negative payoff matrices) for two possible cases: (i) the strategy of company 1 is '*I*', and (ii) the strategy of company 1 is '*NI*'.

The case where the strategy of company 1 is '*I*' is first considered. If all three companies invest in domino prevention, then their costs are just their investment costs, c_i. If (besides company 1) company 2 invests in domino effects prevention, and company 3 does not invest, then companies 1 and 2 incur their investment cost c_i plus the expected loss from a domino effect from company 3 onto respectively company 1 or company 2 (i.e., $P_{3,1}L_1$, respectively $P_{3,2}L_2$). Company 3 just has an expected loss from an internal domino effect, i.e., $P_{3,3}L_3$. If neither company 2 and company 3 invest, then company 1 has an expected loss of its investment costs plus the expected loss from a domino effect from company 2 onto company 1 (i.e., $P_{2,1}L_1$), conditioned on there being no domino effect from company 3 onto company 1 (i.e., times $1-P_{3,1}$), plus the expected loss from a domino effect from company 3 onto company 1 (i.e., $P_{3,1}L_1$), conditioned on there being no domino effect from company 2 onto company 1 (i.e., times $1-P_{2,1}$). The conditions result from the fact that an installation can only explode once (as mentioned before). All the other expected losses composing the cost

Company 3 / Company 2	*I*	*NI*
I	c_2 ; c_3	$c_2 + P_{3,2}L_2; P_{3,3}L_3$
NI	$P_{2,2}L_2; c_3 + P_{2,3}L_3$	$P_{2,2}L_2(1-P_{3,2}) + P_{3,2}L_2(1-P_{2,2});$ $P_{3,3}L_3(1-P_{2,3}) + P_{2,3}L_3(1-P_{3,3})$

Fig. 1. Cost matrix of companies 2 and 3 in the case where the strategy of company 1 is *I*.

Company 3 / Company 2	*I*	*NI*
I	$c_2 + P_{1,2}L_2; c_3 + P_{1,3}L_3$	$c_2 + P_{1,2}L_2(1-P_{3,2}) + P_{3,2}L_2(1-P_{1,2});$ $P_{3,3}L_3(1-P_{1,3}) + P_{1,3}L_3(1-P_{3,3})$
NI	$P_{2,2}L_2(1-P_{1,2}) + P_{1,2}L_2(1-P_{2,2});$ $c_3 + P_{1,3}L_3(1-P_{2,3}) + P_{2,3}L_3(1-P_{1,3})$	$P_{2,2}L_2(1-P_{1,2})(1-P_{3,2})$ $+ P_{1,2}L_2(1-P_{2,2})(1-P_{3,2})$ $+ P_{3,2}L_2(1-P_{1,2})(1-P_{2,2});$ $P_{3,3}L_3(1-P_{1,3})(1-P_{2,3})$ $+ P_{1,3}L_3(1-P_{3,3})(1-P_{2,3})$ $+ P_{2,3}L_3(1-P_{3,3})(1-P_{1,3})$

Fig. 2. Cost matrix of companies 2 and 3 in the case where the strategy of company 1 is *NI*.

matrix are determined in a similar way. The resulting cost matrix can be found in Fig. 1.

In case company 1's strategy is to invest (*I*), choosing *I* is a dominant strategy[2] for company 2 under the conditions that

$$\begin{cases} c_2 < P_{2,2}L_2 \\ c_2 + P_{3,2}L_2 < P_{2,2}L_2(1-P_{3,2}) + P_{3,2}L_2(1-P_{2,2}) \end{cases}$$

Or:

$$\begin{cases} c_2 < P_{2,2}L_2 \\ c_2 < P_{2,2}L_2(1-2P_{3,2}) \end{cases}$$

The first condition is obviously what we would expect to be true in case of a single chemical company deciding whether to invest in domino effects prevention or not, thereby not taking into account the strategies of the other two companies within the cluster of three companies. The second condition, expressing the domino effect risk from a nearby company (thus considering the existence of the cluster in which the company is situated), is evidently stricter than the first condition.

Furthermore, (*I*,*I*) is a Nash equilibrium if $c_i < P_{i,i}L_i$ and is a dominant strategy if $c_i < P_{i,i}L_i(1-2P_{j,i})$ with $i=2$ or 3. (*NI*, *NI*) is a Nash equilibrium if $c_i > P_{i,i}L_i(1-2P_{j,i})$ and is a dominant strategy if $c_i > P_{i,i}L_i$.

For the case where company 1 does not invest (strategy *NI*), a matrix representing the costs incurred by companies 2 and 3 may be determined as well. Fig. 2 illustrates this matrix.

2 A strategy is called 'dominant' if it is the best strategy for every player, independent of what the other players' strategies are.

In the case company 1's strategy is to not invest (*NI*), choosing *I* is a dominant strategy for company 2 under the conditions that

$$\begin{cases} c_2 + P_{1,2}L_2 < P_{2,2}L_2\left(1-P_{1,2}\right) + P_{1,2}L_2\left(1-P_{2,2}\right) \\ c_2 + P_{1,2}L_2\left(1-P_{3,2}\right) + P_{3,2}L_2\left(1-P_{1,2}\right) < P_{2,2}L_2\left(1-P_{1,2}\right) \\ \left(1-P_{3,2}\right) + P_{1,2}L_2\left(1-P_{2,2}\right)\left(1-P_{3,2}\right) + P_{3,2}L_2\left(1-P_{1,2}\right) \\ \left(1-P_{2,2}\right) \end{cases}$$

The conditions for which *I* is a dominant strategy for company 3 can be derived analogously. Furthermore, (I, I) is a Nash equilibrium if $c_i + P_{j,i}L_i < P_{i,i}L_i(1-P_{j,i}) + P_{j,i}L_i(1-P_{i,i})$ with $(i, j) = (2, 1)$ and $(i, j) = (3, 1)$.

Our DE Game is thus characterized by multiple Nash equilibria. Hence, if both the cost matrices from company 1 using '*I*' (Fig. 1) or '*NI*' (Fig. 2) as a strategy are considered, the required conditions to turn the decision of companies 2 and 3 from 'not investing in domino prevention measures' (being part of a Nash equilibrium which is obviously not optimal from a socio-economic viewpoint) to 'investing in domino prevention measures' (being part of a socio-economic optimal Nash equilibrium) can be determined.

The tipping problem is illustrated using the following numerical example. Let $c_1 = 4000€$, $c_2 = c_3 = 700€$. Assume that the probabilities aggregated for all installations within a company and per 10,000 years, are $P_{2,2} = P_{2,3} = P_{2,1} = P_{3,2} = P_{3,3} = P_{3,1} = 0.1$; $P_{1,1} = 0.2$; $P_{1,2} = P_{1,3} = 0.3$. Assume further that the potential company losses are $L_1 = 20{,}000€$ and $L_2 = L_3 = 10{,}000€$.

It is examined what the dominant strategy is for company 1, given that companies 2 and 3 (considered in a sub-cluster of companies) are currently not investing in domino prevention ($\{y\} = \phi$) and do not consider the losses possibly resulting from the other companies in the cluster. In that case the direct losses to companies 2 and 3 are $c_2(\phi) = c_3(\phi) = P_{2,2}L_2(1-P_{1,2})(1-P_{3,2}) = 600€$. It is obvious that, since $c_2 > c_2(\phi)$ and $c_3 > c_3(\phi)$, neither company 2 nor company 3 will invest in domino effects prevention if company 1's strategy is *NI*. Since $c_1(\phi) = P_{1,1}L_1(1-P_{3,1})(1-P_{2,1}) = 3200€$ is smaller than c_1 (=4000€), company 1 will indeed not invest and (*NI*, *NI*, *NI*) is actually a Nash equilibrium. Hence, if company 1 does not invest, then not investing is a dominant strategy for the other companies for all $c_i > 600€$.

Given that the three companies are located within the same cluster, limiting the analysis to direct (internal) domino effects does not offer a solid basis for domino prevention management. Also the indirect loss caused by the fact that only a subset $\{y\}$ of companies is investing in prevention should be considered. An example of how including the indirect risks can alter the analysis, is therefore provided. Assume that company 1 is obliged to invest in domino effects prevention (e.g. due to national or regional regulations and/or legislation) and as a result no negative externality from company 1 is imposed on the other companies (2 and 3). If this is the case, the indirect risk to chemical company *i* (2 or 3) is the same whether company *i* (2 or 3) itself decides to 'invest' or decides to 'not invest', i.e., $l_i(\{y\}, NI) = l_i(\{y\}, I)$ (where $i = 2, 3$), hence we can use expression

$$\tilde{c}_i(\{y\}) = P_i \cdot \left(L_i \prod_{j \neq i, j \in \{x/y\}} \left(1-P_j\right) - l_i(\{y\}, NI) \right)$$

to determine the critical cost levels of companies 2 and 3, which in both cases amount to 800€.

Since both c_2 and c_3 are smaller, companies 2 and 3 will be changing their strategy of *NI* to *I* as a result of company 1 doing so. Therefore, company 1 has the power to tip the other companies within the three-company cluster from one strategy to another strategy. In other words, one company's strategic choice concerning domino prevention may significantly influence the other companies' domino prevention-related strategic decisions in a chemical cluster. This is an important finding implying that, e.g. company-specific incentives or well-elaborated domino prevention regulations can lead to substantial safety improvements within chemical clusters.

Besides the obvious advantages of the game-theoretical approach elaborated in this paper for improving safety in chemical clusters by possibly increasing the investments in domino effects prevention, another possible important implementation of these findings might be situated in the security field. Further research would have to be carried out to verify the suggested model's usefulness as regards the prevention of incidents intentionally designed to damage.

Future research is required to validate the findings provided by the suggested theoretical model in this article and an industrial case-study needs to be investigated.

5. Conclusions

Although there appear to be limitless possibilities as to what might go wrong in a chemical cluster and what the industry might have to do to protect against cross-plant escalation effects, this article offers some exciting new insights from a game-theoretical viewpoint into tackling this very complex problem. Players (chemical companies) in a chemical cluster may end up playing a Nash equilibrium (i.e., a win–win situation of strategic decisions) from the perspective that academic research can help them assessing the equilibrium's consistency or robustness, or otherwise help them identify the appealing possibilities at their disposal to reach an agreement on playing such equilibrium. This observation is particularly interesting in our domino effects prevention case, where in real industrial settings only partial protection is provided by single plant investments, and where these investments are strategically complementary. If it is possible to change the strategic choice of a small number of players (companies) of the cluster, it might be possible this way to tip all the rest of the players within the cluster to change their strategies from a socially non-optimal situation to a socially optimal situation. If academic research would lead to identification of those chemical plants being very influential (and belonging to the tipping inducing sub-cluster) and industrial associations or authorities might be able to persuade these plants into changing their strategic positions, safety and/or security might truly be optimized within a chemical industrial area.

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